A Technique for Measuring Phase Modulation or Rapid Phase Changes of a Microwave Signal

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Abstract—The system takes advantage of the fact that a phase-modulated carrier signal will produce equivalent frequency modulation, where the change of frequency is proportional to the time rate of change of the phase variation. A mirowave carrier frequency, FC, is shifted in a single sideband generator to a frequency, FC+FM. The upper sideband is transmitted through the phase modulating medium and then mixed with the original carrier frequency. The difference frequency FM contains the phase modulation information and, after preamplification, the signal is put through a wide-band limiter-discriminator of center frequency, FM. The discriminator output voltage will be directly proportional to the rate of phase change. To compensate for the time rate of change dependency, the discriminator output is followed by an integrating network ($E_0 \sim 1/f$) which will produce an output voltage proportional to the phase change only. Calibration is accomplished by imposing a known amount of phase modulation on the sideband modulating signal FM and observing the system output signal. This measuring scheme has been built in the 70 Gc/s band and is capable of following a wide range of phase changes over microsecond periods.

I. Introduction

MANY SCHEMES have been devised to detect phase shift at microwave frequencies. Most of them are capable of high accuracy when measuring a slowly changing phase. The problems become more complex when one desires to monitor phase changes which take place in the order of a few microseconds. The double-probe method [1] has been adapted to measure phase modulation, but falls short in that the range of phase change is limited, i.e., up to 90°, and that the technique is difficult to adapt to millimeter microwave measurements. The "zebra-stripe" Wharton [2], [3], microwave interferometer is capable of handling wide dynamic phase shifts, but a large number of fringe shifts, due to phase changes, are cumbersome to resolve with this technique. It is also difficult to resolve rapid phase changes in the low microsecond range. Another system developed by Osborne [4] presents phase vs. time as polar plot, where timing pulses appear as dots on the display. Multiradian phase changes are again difficult to resolve.

The system to be described is modeled after one suggested by Heald [5] but differs in that the carrier frequency is CW and a method of calibration has been incorporated.

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First we shall discuss the theoretical aspects of the scheme. The operation of the system will be explained, followed by a detailed description of the principal components of the measuring scheme. Then the system performance, data, analysis of the data and conclusions are reported. Finally, a brief discussion on the application of the system in the measurement of plasma electron density will conclude the paper.

A. Theoretical Considerations

It will be helpful to review some of the concepts of a phase-modulated signal and to show how equivalent frequency modulation is generated. We also shall consider bandwidth requirements for the range of phase modulation to be measured.

For a phase-modulated signal, the instantaneous value of the modulated carrier voltage may be written

$$e = E_0 \sin [\omega t + A(t)] = E_0 \sin \phi$$

where $A(t)$ is the phase modulating component which is a function of time. The apparent instantaneous frequency, f_t, is the time derivative of the phase ϕ.

$$f_t = \frac{1}{2\pi}\frac{d\phi}{dt} = \frac{\omega}{2\pi} + \frac{1}{2\pi}\frac{d}{dt}A(t)$$

$$= f + \frac{1}{2\pi}\frac{d}{dt}A(t)$$

where f = CW signal frequency, and $(1/2\pi)(d/dt)A(t)$ = the change of instantaneous frequency due to the phase modulation. For sinusoidal phase modulation $A(t) = A \sin \omega_1 t$, and $(d/dt)A(t) = \omega_1 A \cos \omega_1 t$.

The instantaneous frequency shift then is

$$\Delta f = \frac{1}{2\pi}\frac{d}{dt}A(t) = f_1 A \cos \omega_1 t$$

where f_1 is the frequency of the phase modulating process.

It is to be noted that Δf is proportional not only to A, the peak phase change, but also to f_1. Hence, when the phase-modulated signal is passed through a frequency discriminator circuit, the faster the rate of change of phase, the larger Δf, and the greater the voltage developed by the discriminator detector network. If the discriminator is followed by an integrating network, wherein $E \sim 1/f$, the resulting output voltage will be proportional to the amplitude of the phase change only, and not to the rate of change.

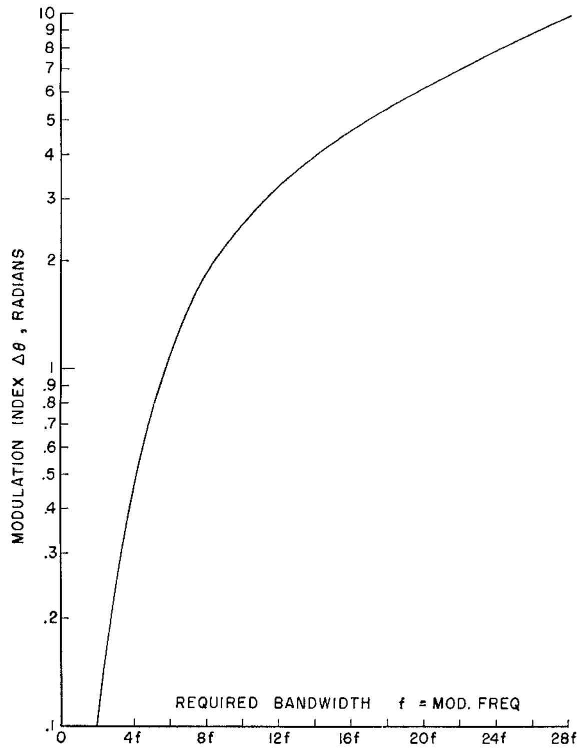


Fig. 1. Modulation index vs. required bandwidth.

B. Bandwidth Requirements

One of the fundamental differences between a frequency-modulated (FM) signal and phase-modulated (PM) signal is the spectrum distribution about the carrier frequency. To illustrate, the basic relationships between modulation index, number of sidebands, and bandwidth are tabulated.

Basic relationships: PM and FM

1) Number of sidebands$\sim$modulation index.
2) Bandwidth$\sim$number of sidebands by f_1.
3) Bandwidth$\sim$modulation index by f_1.

Type of modulation	Modulation index	Bandwidth
FM	$\Delta f/f_1$	Δf
PM	$\Delta\theta$	$f_1\Delta\theta$

where Δf = peak change in carrier frequency; $\Delta\theta$ = peak phase swing of carrier; f_1 = modulating frequency.

Thus, for FM, the number of sidebands will vary with f_1 for a fixed Δf, but the bandwidth is approximately constant. However, for PM the number of sidebands is constant for a fixed $\Delta\theta$, but the bandwidth will vary directly with f_1. Therefore, when estimating bandwidth requirements for the preamplifier and limiter-discriminator in a phase measuring system, both the maximum phase change and the rate of phase change must be considered.

From a table by Hund [6], we have plotted the graph in Fig. 1 which relates required bandwidth to modulation index. This graph is used to predict the system performance of the limiter-discriminator and calibrating sources which are described in a later section.

II. System Operation

Figure 2 shows a block diagram of the system which has been built to measure the phase modulation of a 70 Gc/s signal. The power at the carrier frequency FC is divided into two paths by a 10-dB directional coupler. One coupler output goes to a single sideband generator where the carrier is shifted to FC+30 Mc/s. The upper sideband is passed through the phase modulating medium and then on to a balanced mixer. The local oscillator input to the mixer is the original carrier FC from the 10-dB down port of the directional coupler. The 30 Mc/s difference frequency output of the mixer is amplified in a conventional wideband preamplifier and then applied to a limiter-discriminator circuit which has a linear "S" curve over ± 3 Mc/s. The discriminator detector output is followed by an integrating operational amplifier, wherein the RC time constant of the integrator is consistent with the rate of phase change to be measured. The integrator voltage, being the final output desired, is then applied to the Y axis of an oscilloscope which is triggered by a timing pulse.

To calibrate the integrator output signal, a scheme is used to phase modulate the 30 Mc/s input to the single sideband generator. Up to 3π radians of linear phase modulation may be electrically imposed at rates up to 10 kc/s. Higher rates of phase change may be used at the expense of lower peak phase changes due to bandwidth limitations in the phase modulator.

A. System Components

Figure 3(a) and (b) are pictures of the test set. The individual parts of the measuring scheme are treated in detail.

Waveguide Circuitry: The waveguide circuit is made up of standard, commercially available components in RG-98/u size waveguide. Figure 3(b) is a photograph of the waveguide section showing the single sideband generator and the balanced mixer. The single sideband generator has been the subject of another paper [7] and we simply present its performance data here:

Carrier input power:	20 mW
30 Mc/s modulation power:	10 mW
Carrier and unwanted: to upper sideband	>20 dB down from wanted sideband
Conversion loss-carrier to upper sideband	20 to 25 dB

The isolator between the directional coupler and single sideband generator suppresses the reflected carrier and unwanted (lower) sideband traveling back towards the generator. This feature is important because when the attenuation in the upper sideband transmission path is in the order of 30 to 40 dB, any unwanted sideband signal coupling through the 10-dB coupler into the balanced mixer will cause severe amplitude variations of the IF signal as the phase of the upper sideband is varied.

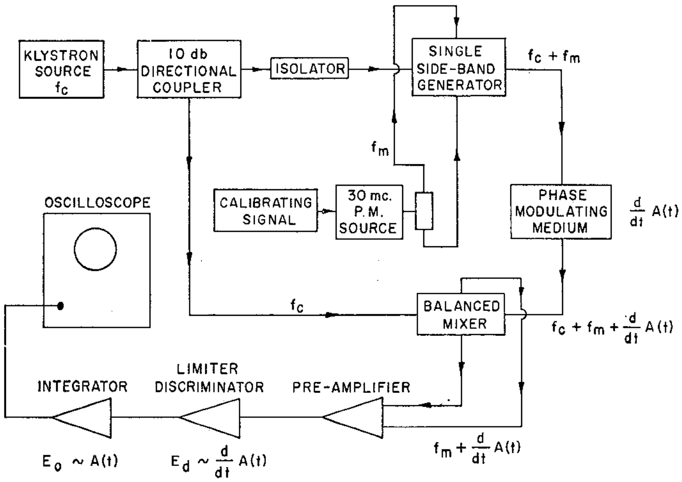
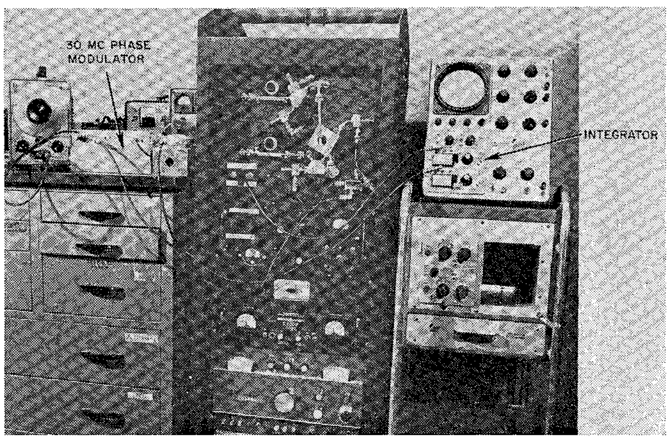
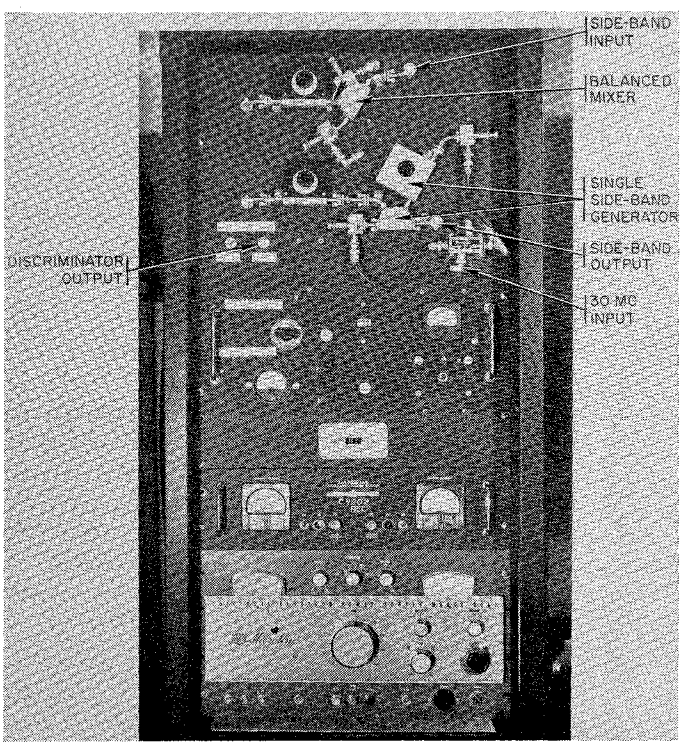


Fig. 2. Block diagram of PM measuring system.



(a)



(b)

Fig. 3. (a) Laboratory setup of microwave phase measuring system. (b) Test set console showing waveguide network.

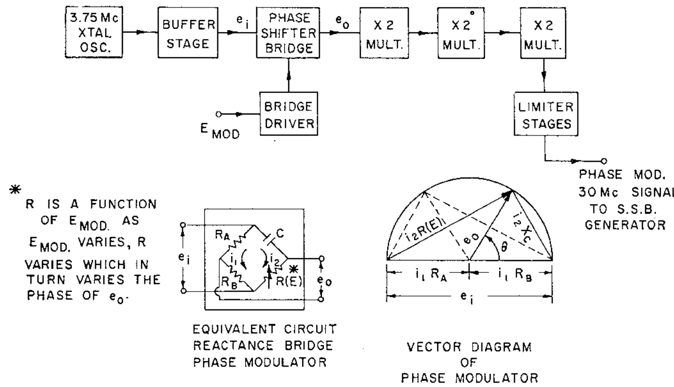


Fig. 4. Block diagram of 30 Mc/s phase modulator.

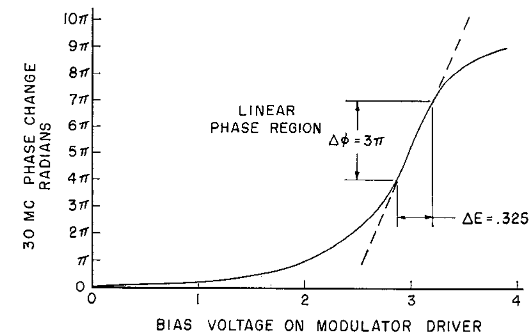


Fig. 5. Calibration of phase modulator.

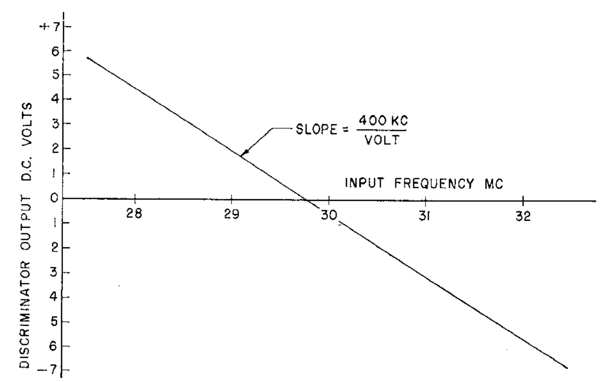


Fig. 6. Discriminator calibration.

30 Mc/s Phase Modulator: Figure 4 shows, in block diagram form, the 30 Mc/s phase modulator [8] which was used to calibrate the system. Briefly, a 3.75 Mc/s crystal-controlled oscillator is phase-modulated in a reactance bridge phase shifter by an external calibrating signal. This phase-modulated voltage then goes through a series of multiplier stages wherein the frequency is translated up to 30 Mc/s, and the phase swing is also multiplied by 8. Limiting stages after the last multiplier are required to minimize the amplitude modulation of the phase-modulated signal. An output voltage of 2 volts rms is available to drive a 50-ohm load. A plot of output phase shift vs. bias voltage on the phase-modulated stage is shown on Fig. 5 which indicates a linear phase swing of 3π radians is available for calibration purposes. The 3-dB bandwidth of the 30 Mc/s stages is

3 Mc/s and, as previously shown, this factor must be taken into account when using high-frequency calibration signals.

IF Preamplifier: The preamplifier is one of standard design made to match the crystals in the balanced mixer. A 3-dB bandwidth of 10 Mc/s, and a gain of 35 dB provides adequate preamplification of the phase-modulated 30 Mc/s IF signal.

Limiter-Discriminator: The limiter-discriminator is a wide bandwidth centered at 30 Mc/s. The discriminator "S" curve has a linear portion over ± 3 Mc/s. A plot of this curve is shown in Fig. 6. A 5 Mc/s video amplifier and cathode follower are included in the unit. Since the output impedance is in the order of 75 ohms, it makes it possible to locate the integrating operational amplifier remotely from the rest of the system.

Integrator: The integrator plays a primary role in the overall system performance. An operational amplifier, whose input impedance is resistive and feedback element is a capacitor, will act as an integrator providing the RC time constant is consistent with the phase modulation rates to be measured. The particular amplifier used is a type "0" plug-in assembly for a Tektronix Oscilloscope. There are facilities for varying both R and C over wide ranges as well as using external elements if desired. In general, the RC time constant of the integrator should be approximately equal to the lowest period expected in the rate of phase change. One characteristic of this type of integrating amplifier is that the low-frequency gain is many times greater than the high-frequency gain. It is, therefore, important that there be a minimum of low-frequency ripple and hum present in the video amplifier output stage in order to avoid excessive ripple on the integrator output voltage. In Section III the range of frequencies is given for various RC time constants, upon which the operation amplifier performs as an integrator.

III. System Performances

A variety of oscilloscope measurements have been made to evaluate the system performance. The following sections outline the testing procedure used and the data taken on each of the functions tested. Figure 7 shows a block diagram of the test setup indicating the test points.

A. Discriminator Performance

The following procedure was used in checking the discriminator.

1) A triangular shape voltage was applied to the 30 Mc/s phase modulator input. The peak-to-peak amplitude produced a linear phase modulation of 2π rad on the 30 Mc/s modulation signal.
2) The direct output of the discriminator was observed through an electronic low-pass filter, FC = 200 kc/s. The use of the filter enabled clearer pictures of the discriminator output waveform. In normal use, the filter is not needed since the integrator inherently eliminated the "white noise" which is on the desired signal.

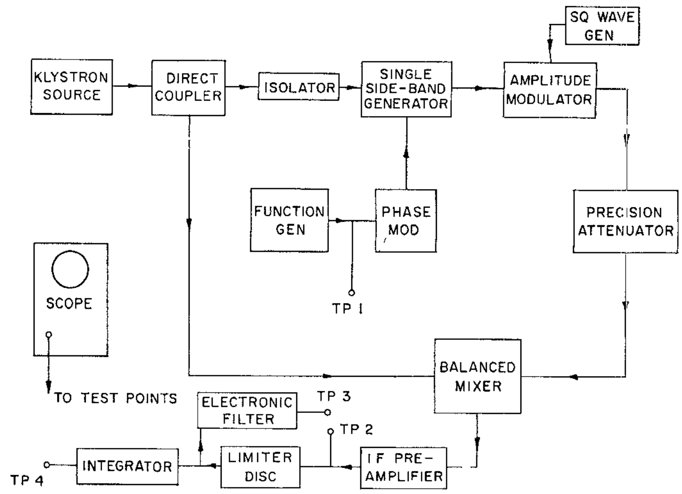


Fig. 7. Block diagram of setup for testing system.

From the theoretical considerations, we know that the instantaneous change in the microwave signal frequency due to the phase modulation is

$$\Delta f = \frac{1}{2\pi} \frac{d}{dt} A(t)$$

where $A(t)$ represents the phase modulation caused by the triangular voltage into the phase modulator.

Two rates of phase change were used:

$$\text{Case a)} \quad \frac{d}{dt} A(t) = \frac{2\pi \text{ rad}}{50 \ \mu\text{s}}$$

$$\text{Case b)} \quad \frac{d}{dt} A(t) = \frac{2\pi \text{ rad}}{25 \ \mu\text{s}}$$

Thus

$$\Delta f_a = \frac{1}{2\pi} \times \frac{2\pi}{50 \times 10^{-6}} = 20 \text{ kc/s}$$

$$\Delta f_b = \frac{1}{2\pi} \times \frac{2\pi}{25 \times 10^{-6}} = 40 \text{ kc/s}$$

To calculate the voltage out of the discriminator for a given change of frequency, we refer to Fig. 6 which shows the discriminator sensitivity in volts per kc/s.

$$S_d = \frac{1 \text{ volt}}{400 \text{ kc/s}}$$

The expected discriminator voltages for Cases a) and b) are:

$$\begin{aligned} E_a &= \Delta f_a \times S_d \\ &= 20 \text{ kc/s} \times \frac{1 \text{ volt}}{400 \text{ kc/s}} \\ &= 50 \text{ mV} \end{aligned}$$

Similarly,

$$E_b = \Delta f_b \times S_d = 100 \text{ mV}$$

Shown in Fig. 8 are the oscilloscope tracings which were taken at TP1 and TP3 under the foregoing conditions. The measured gain of the electronic low-pass filter was found to be 1.9, and this factor must be used when calculating the actual voltage out of the discriminator. We have tabulated the calculated and measured test data.

Peak-to-peak phase change radians	dt μs	Discriminator output voltage mV		
		Calc.	Meas. TP3	Meas. $\div 1.9$
2π	50	50	90	47.5
2π	25	100	180	95.0

The results verify two of the theoretical considerations, namely:

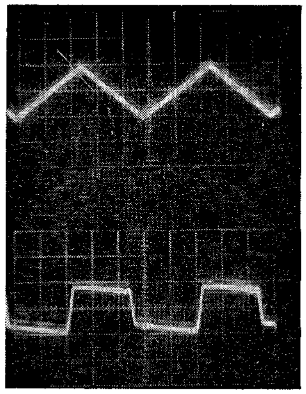
1) The discriminator output waveform is the differential form of the original phase modulating wave shape.
2) If the rate of phase change is increased, then the output voltage of the discriminator is increased proportionately.

B. Integrator Performance

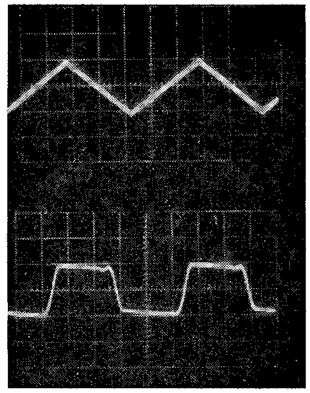
The primary purpose of this test is to show that the output voltage of the integrator is proportional to the magnitude of the phase modulation and independent of the rate of phase change.

The input phase modulating waveform, a TP1, was adjusted to produce a peak-to-peak linear phase change of 2π rad. The rate of change was varied by using repetitive rates of 5, 10, and 20 kc/s. For each frequency, the output of the integrator at TP4 was recorded. Figure 9 is the oscilloscope reproduction of input and output waveforms under the preceding conditions. We note that the peak-to-peak integrator output is constant, and thus independent of the time rate of phase change. It is important to stress at this time that rate independency is only true over a range where the RC time constant of the integrating operational amplifier is consistent with the rates of phase change. In other words, for slower rates of phase change the RC time constant would need to be increased. Generally, as an initial setting, it is recommended that the RC time constant be set equal to the slowest rate of change expected. The following test was performed to establish the actual operating range for different RC time constants.

A constant peak-to-peak voltage was applied to the phase modulator which caused a peak-to-peak phase swing of one radian. The frequency of the modulating voltage was varied over the range of 1 to 250 kc/s. Four



(a)



(b)

Fig. 8. Discriminator test waveforms: *top trace*—phase modulator voltage; *bottom trace*—discriminator output voltage.

Fig.	Sweep Speed μs/cm	Linear Phase Change	μs	Volts/cm	
				Bottom	Top
(a)	20	2π	50	0.05	0.10
(b)	10	2π	25	0.10	0.10

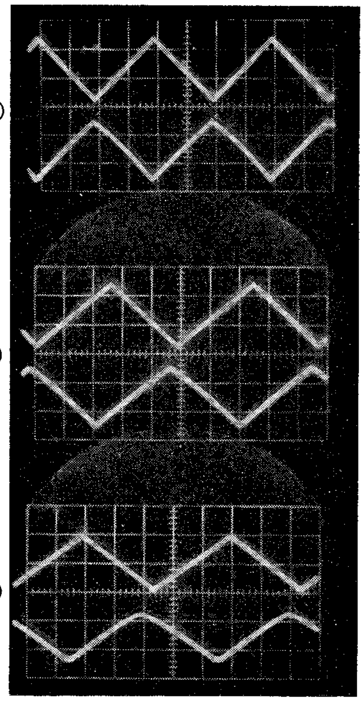


Fig. 9. Integrator test waveforms: *top trace*—phase modulator input; *bottom trace*—integrator output.

Fig.	Sweep Speed μs/cm	Linear Phase Change	dt μs	Volts/cm		Integrator RC
				Top	Bottom	
(a)	50	2π rad	100	0.1	0.05	$R=10$ kΩ
(b)	20	2π rad	50	0.1	0.05	
(c)	10	2π rad	25	0.1	0.05	$C=0.001$ μF

different RC time constants were set on the type "0" operational amplifier, and the peak-to-peak output voltage of the integrator was recorded. Table I summarizes the test data.

TABLE I
INTEGRATOR OUTPUT VS. FREQUENCY

$RC\rightarrow$ freq. kc/s	Peak-to-peak integrator output voltage				
	10 kΩ 10 pF	10 kΩ 0.0001 μf	10 kΩ 0.001 μf	10 kΩ 0.01 μf	
1	0	0.2	0.1	0.015	
3	1.0	1.5	0.14	0.015	
5	2.6	2.4	0.13	0.015	
10	22.0	1.5	0.13	0.015	Recommended
50	12.0	1.3	0.13	0.015	operating
100	12.0	1.3	0.13	0.015	range
250	11.0	1.2	0.11	0.015	

It should be noted that the advantage of the wide range handled by the highest RC time constant has the disadvantage of low-output voltage, and also the lowest signal-to-noise ratio.

C. Response to "Fine Grain" Phase Modulation

To illustrate the response of the system to a combination of slow and fast rates of phase change, the linear phase change caused by a triangular waveform had superimposed on it a small sine wave. The ratio of the slow to fast rates of change was adjusted for 1 to 10. Figure 10 is the resulting oscilloscope picture showing input and output waveforms. We note that the discriminator-integrator scheme does reproduce to original modulating wave shape.

D. Effect of Amplitude Modulation

In general, it is expected that a certain degree of amplitude modulation will accompany the phase-modulated signal that is being detected. For the purpose of demonstrating the response of the limiter-discriminator to an AM-PM signal, the following test was performed. A triangular wave was applied to the phase modulator, and at the same time a ferrite amplitude modulator was inserted in the upper sideband signal transmission path. Figure 11 shows the oscilloscope tracings that were made of the integrator output with and without AM, and also of the amplitude-modulated IF frequency at TP2. The amount of AM was adjusted to approximately 50 per cent. Beyond this amount, it was found that the ferrite modulator itself would cause a noticeable amount of phase modulation.

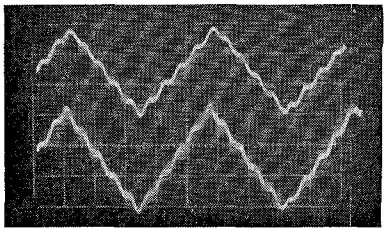


Fig. 10. Oscilloscope picture showing system response to "fine-grain" variation of phase. *Top trace*—phase modulator input; *bottom trace*—integrator output.

Sweep speed—20 μs/cm
Sawtooth —10 kc $\Delta\theta \approx 3\pi$ rad
Sinewave —100 kc $\Delta\theta \approx \pi/10$ rad

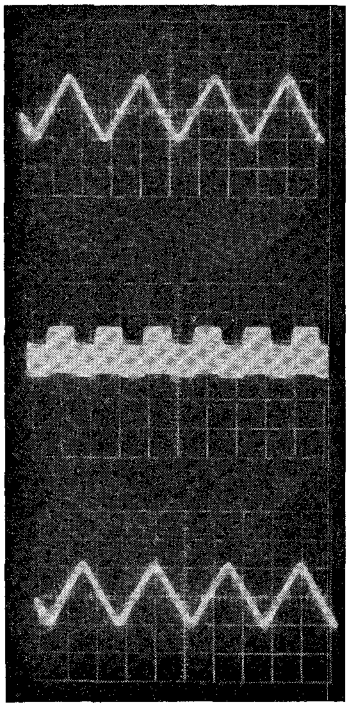


Fig. 11. Phase modulation plus amplitude modulation. *Top trace*—integrator output PM. Sweep 50 μs/cm; *bottom trace*—integrator output PM+AM; *middle trace*—30 Mc/s. IF signal showing 3 kc/s square wave modulation. Sweep 200 μs/cm.

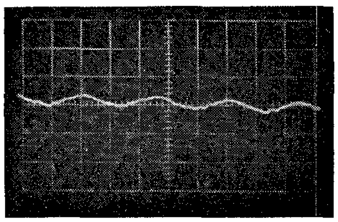


Fig. 12. Minimum sensitivity test, showing integrator output for a peak-to-peak phase modulation of $\pi/100$ rad. Sweep speed 20 μs/cm.

Vertical sensitivity = 0.05 V/cm
$R = 10$ kΩ $C = 0.011$ μF

E. Minimum Sensitivity

An oscilloscope tracing was made of the integrator output under the condition of a peak-to-peak phase swing of $2\pi/100$ rad or approximately 4°. The resulting reproduction, Fig. 12, indicates a relatively good signal-to-noise ratio even at very low-phase modulation amplitudes.

The system sensitivity to small phase changes will vary depending upon (a) the time rate of phase change, and (b) the setting of the RC time constant on the integrator. Since the integrator gain varies as $1/f$, the low-frequency noise and hum at the discriminator output will be emphasized for the larger RC time constants which are necessary for the slower time rates of phase change.

IV. Conclusions

The system that has been described in the preceding sections may be analyzed for its advantages and disadvantages.

A. Advantages

1) Response to rapid phase changes.
2) Response to multiradian phase changes, i.e., greater than 2π rad.
3) Insensitive to amplitude variations of signal within the range of the 30 Mc/s limiter.
4) System output voltage is directly proportional to phase change.

B. Disadvantages

1) Response to a variable time rate of phase change limited by the integrating operational amplifier.
2) Means must be provided to calibrate the integrator output of the system.
3) Large phase shifts in very short, microsecond periods are limited by the inherent finite bandwidth of limiter-discriminator.
4) The high gain of the integrating operational amplifier at low frequencies limits the minimum phase changes that may be observed at relatively slow rates of phase change, i.e., in the millisecond range.

C. Application

The particular application for this system has been in the measurement of the electron density in a hot plasma, and of special interest rapid density variations. It is a well-known fact that the phase of a microwave probing signal will be changed because of the varying index of refraction of the plasma medium. In a pulse-type plasma device, the electron density may vary rapidly with time, depending upon the experimental conditions. The system described herein has been compared to the Wharton [2], [3], or zebra-stripe system in respect to their output displays. Both systems were set up to use a common pair of waveguide horns between which the plasma medium was present. Figure 13 illustrates the resulting oscilloscope tracings of the output information of the system. Frames (a) and (c) are the Wharton system, and (b) and (d) are the phase discriminator system. In the slower sweep-speed frames, it is evident that the multiradian phase change is easily distinguished in the phase discriminator output. The fast sweep displays show the ability of the phase discriminator to present the "fine-structure" more clearly than the zebra-stripe reproduction. It should be pointed out that the discriminator system performance is not satisfactory when the plasma density is high enough to cut off the microwave transmission. The high-noise output of the system under the cutoff condition produces false signals at the integrator output.

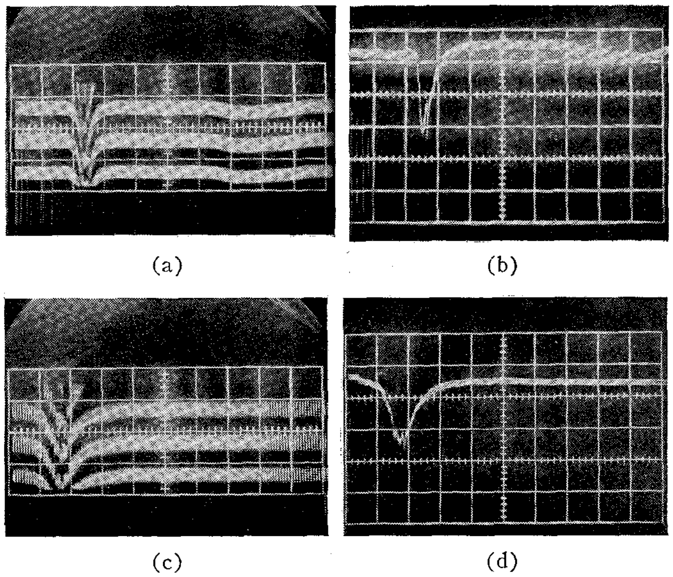


Fig. 13. Oscillosocope pictures comparing "zebra-stripe" and phase discriminator systems. (a) and (b) sweep speed—200 μs/cm; (c) and (d) sweep speed—50 μs/cm.

We may say that the phase discriminator system is not a complete substitute for the zebra-stripe system, but should prove valuable when looking for rapid variations in electron density.

References

[1] Lacy, P., A versatile phase measurement method for transmission line networks, *IRE Trans. on Microwave Theory and Techniques*, vol MTT-9, Nov 1961, pp 568–569.
[2] Wharton, C. B., Microwave diagnostics for controlled fusion research, UCRL Rept. no. 4836, 1957.
[3] Ernst, W. P., An electron density measuring system for hot plasma research, *Microwave J.*, vol 4, no. 2, Feb 1961.
[4] Osborne, F. L., Microwave plasma diagnostics by phase and amplitude measurements, RCA Rept. no. 9-801-15, Montreal, Que., Canada.
[5] Heald, M. A., Proposal for a new type of phase measuring system having an output proportion to phase shift, PPL Microwave Memo no. 4, Princeton University, Princeton, N. J., 1959.
[6] Hund, A., *Frequency Modulation*, 1st ed. New York: McGraw-Hill, 1942, p 33.
[7] Ernst, W. P., and A. Skislak, Millimeter single side-band generators, MATT-167, Plasma Physics Lab., Princeton University, Princeton, N. J.
[8] Weissenberger, A. W., Internal communication.
[9] Seely, S., *Electronic Engineering*, New York: McGraw-Hill, 1956, pp 169–176.
[10] Sparks, R. A., Microwave phase measurements, *Microwaves*, Jan 1963.
[11] Terman, F. E., and J. M. Pettit, *Electronic Measurements*, New York: McGraw-Hill, 1952, pp 267–275.